Formulaire PCSI-MPSI.
Fonctions trigonométriques et hyperboliques.

$$\begin{aligned}
\cos(a+b) &= \cos a.\cos b - \sin a.\sin b \\
\cos(a-b) &= \cos a.\cos b + \sin a.\sin b \\
\sin(a+b) &= \sin a.\cos b + \sin b.\cos a \\
\sin(a-b) &= \sin a.\cos b - \sin b.\cos a \\
\tan(a+b) &= \frac{\tan a + \tan b}{1 - \tan a.\tan b} \\
\tan(a-b) &= \frac{\tan a - \tan b}{1 + \tan a.\tan b}
\end{aligned}$$

$$\begin{aligned}
\cos 2a &= 2.\cos^2 a - 1 \\
&= 1 - 2.\sin^2 a \\
&= \cos^2 a - \sin^2 a \\
\sin 2a &= 2.\sin a.\cos a \\
\tan 2a &= \frac{2\tan a}{1-\tan^2 a}
\end{aligned}$$

$$\begin{aligned}
\cos a.\cos b &= \frac{1}{2}[\cos(a+b) + \cos(a-b)] \\
\sin a.\sin b &= \frac{1}{2}[\cos(a-b) - \cos(a+b)] \\
\sin a.\cos b &= \frac{1}{2}[\sin(a+b) + \sin(a-b)]
\end{aligned}$$

$$\begin{aligned}
\cos p + \cos q &= 2.\cos\frac{p+q}{2}.\cos\frac{p-q}{2} \\
\cos p - \cos q &= -2.\sin\frac{p+q}{2}.\sin\frac{p-q}{2} \\
\sin p + \sin q &= 2.\sin\frac{p+q}{2}.\cos\frac{p-q}{2} \\
\sin p - \sin q &= 2.\sin\frac{p-q}{2}.\cos\frac{p+q}{2}
\end{aligned}$$

$$\begin{aligned}
\mathrm{ch}(a+b) &= \mathrm{ch}a.\mathrm{ch}b + \mathrm{sh}a.\mathrm{sh}b \\
\mathrm{ch}(a-b) &= \mathrm{ch}a.\mathrm{ch}b - \mathrm{sh}a.\mathrm{sh}b \\
\mathrm{sh}(a+b) &= \mathrm{sh}a.\mathrm{ch}b + \mathrm{sh}b.\mathrm{ch}a \\
\mathrm{sh}(a-b) &= \mathrm{sh}a.\mathrm{ch}b - \mathrm{sh}b.\mathrm{ch}a \\
\tanh(a+b) &= \frac{\tanh a + \tanh b}{1 + \tanh a.\tanh b} \\
\tanh(a-b) &= \frac{\tanh a - \tanh b}{1 - \tanh a.\tanh b}
\end{aligned}$$

$$\begin{aligned}
\mathrm{ch}2a &= 2.\mathrm{ch}^2 a - 1 \\
&= 1 + 2.\mathrm{sh}^2 a \\
&= \mathrm{ch}^2 a + \mathrm{sh}^2 a \\
\mathrm{sh}2a &= 2.\mathrm{sh}a.\mathrm{ch}a \\
\tanh 2a &= \frac{2\tanh a}{1+\tanh^2 a}
\end{aligned}$$

$$\begin{aligned}
\mathrm{ch}a.\mathrm{ch}b &= \frac{1}{2}\left[\mathrm{ch}(a+b) + \mathrm{ch}(a-b)\right] \\
\mathrm{sh}a.\mathrm{sh}b &= \frac{1}{2}\left[\mathrm{ch}(a+b) - \mathrm{ch}(a-b)\right] \\
\mathrm{sh}a.\mathrm{ch}b &= \frac{1}{2}\left[\mathrm{sh}(a+b) + \mathrm{sh}(a-b)\right]
\end{aligned}$$

$$\begin{aligned}
\mathrm{ch}p + \mathrm{ch}q &= 2.\mathrm{ch}\frac{p+q}{2}.\mathrm{ch}\frac{p-q}{2} \\
\mathrm{ch}p - \mathrm{ch}q &= 2.\mathrm{sh}\frac{p+q}{2}.\mathrm{sh}\frac{p-q}{2} \\
\mathrm{sh}p + \mathrm{sh}q &= 2.\mathrm{sh}\frac{p+q}{2}.\mathrm{ch}\frac{p-q}{2} \\
\mathrm{sh}p - \mathrm{sh}q &= 2.\mathrm{sh}\frac{p-q}{2}.\mathrm{ch}\frac{p+q}{2}
\end{aligned}$$

$$\text{avec } t=\tan\frac{x}{2}\begin{cases}\cos x &= \frac{1-t^2}{1+t^2}\\ \sin x &= \frac{2t}{1+t^2}\\ \tan x &= \frac{2t}{1-t^2}\end{cases}\qquad \text{avec } t=\tanh\frac{x}{2}\begin{cases}\mathrm{ch}x &= \frac{1+t^2}{1-t^2}\\ \mathrm{sh}x &= \frac{2t}{1-t^2}\\ \tanh x &= \frac{2t}{1+t^2}\end{cases}$$

$$\cos' x = -\sin x \qquad \mathrm{ch}'x = \mathrm{sh}x$$

$$\sin' x = \cos x \qquad \mathrm{sh}'x = \mathrm{ch}x$$

$$\tan' x = 1+\tan^2 x = \frac{1}{\cos^2 x} \qquad \tanh' x = 1-\tanh^2 x = \frac{1}{\mathrm{ch}^2 x}$$

$$\mathrm{cotan}'x = -1-\mathrm{cotan}^2 x = \frac{-1}{\sin^2 x} \qquad \coth' x = 1-\coth^2 x = \frac{-1}{\mathrm{sh}^2 x}$$

$$\mathrm{Arccos}'x = \frac{-1}{\sqrt{1-x^2}} \quad (|x|<1) \qquad \mathrm{Argch}'x = \frac{1}{\sqrt{x^2-1}} \quad (x>1)$$

$$\mathrm{Arcsin}'x = \frac{1}{\sqrt{1-x^2}} \quad (|x|<1) \qquad \mathrm{Argsh}'x = \frac{1}{\sqrt{x^2+1}}$$

$$\mathrm{Arctan}'x = \frac{1}{1+x^2} \qquad \mathrm{Argth}'x = \frac{1}{1-x^2} \quad (|x|<1)$$

$$\mathrm{Arccotan}'x = \frac{-1}{1+x^2} \qquad \mathrm{Argcoth}'x = \frac{1}{1-x^2} \quad (|x|>1)$$